# POWER TRANSMISSION IN MICROWAVE SYSTEMS

**or**

***What the Hell is Going on Between Magnetron and Load!***

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# 1. INTRODUCTION

Writing of this text was motivated by the known although in some situations rather unexpected phenomenon that in a microwave system consisting of a signal source, a transmission line, and a load, the power carried by the wave travelling toward load may be greater than the available signal source power. Specifically, in high-power industrial applications, the power supplied by a magnetron can be computed from the anode voltage, anode current, and the device efficiency. The power carried by the wave travelling to the load can be measured using a directional coupler. It was observed that this power may be several times higher than the power available from the magnetron. This could lead to mistrust to measurement devices (couplers) and measurement methods. The paper attempts:

- To clear out the situation for sake of those who are not specialists in the field of microwaves (but rather users of the technology) by providing an explanation to the observed effects.
- To suggest what measurements should be actually taken to the satisfaction of the user and what their accuracy limitations are.

The **PowTrans** program (Microwave Power Transmission Calculator) has been developed, supplementing the theory and enabling a variety of useful simulations. Except aiding the study it can be used to make practical assessments in real situations.

# 2. STUDIED SYSTEM

The studied system consists of three basic blocks (Fig. 1):

- Load (working space)
- Transmission medium (transmission line, waveguide)
- Signal source (magnetron)

The circuit-theory representation of the system is shown in Fig. 2. In our further discussions, we shall assume the system linear, the transmission medium lossless, and the signals harmonic.

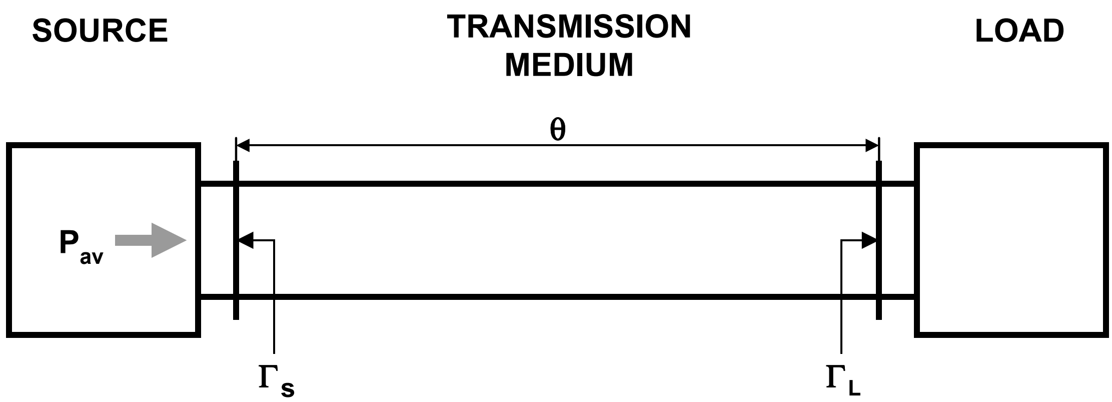


Fig. 1: The studied system

## *2.1 Load*

A load is characterized by its (complex) impedance $Z_L$ or, equivalently, by its reflection coefficient

$$\Gamma_L = |\Gamma_L| \exp(j\varphi_L)$$

The two quantities are related by the equation

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{1}$$

where $Z_0$ is an arbitrary real *reference impedance*. It is meaningful to choose the reference impedance to be equal to the characteristic impedance of the interconnecting transmission line.

If a wave travelling in the transmission line is incidend on the load, a part of the wave energy is reflected back; the other part is transmitted to the load. If the incident wave carries the power $P_i$, the reflected power is

$$P_r = P_i \left|\Gamma_L\right|^2 \tag{2}$$

and the transmitted (absorbed) power is

$$P_L = P_i \left(1 - \left|\Gamma_L\right|^2\right) = P_i m_L \tag{3}$$

where

$$m_L = 1 - \left|\Gamma_L\right|^2 \tag{4}$$

is termed *load mismatch factor*.

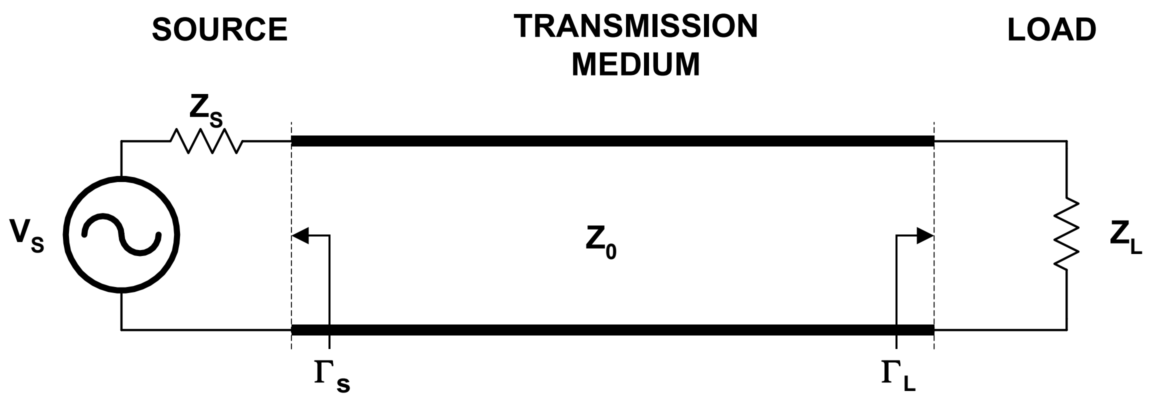


Fig. 2: Circuit representation of the studied system

## *2.2 Transmission Medium*

Generally speaking, transmission medium is an arbitrary circuit connecting the source and the load. A linear circuit is fully characterized by four scattering parameters. In many practical cases, the transmission medium is a section of homogeneous transmission line or waveguide with negligible losses. For sake of text simplicity, we shall often refer to transmission medium as to waveguide.

Basic parameters characterizing such transmission medium (and from which the scattering parameters can be derived) are:

<u>Characteristic impedance</u> $Z_0$. In case of waveguides, the definition of $Z_0$ is ambiguous and in many cases (including our case) we can put $Z_0$=1.

<u>Electrical length</u> θ, expressed in angular units (radians), defined as

$$\theta = 2\pi L / \lambda \tag{5}$$

where $L$ is the physical length of the waveguide and λ the wavelength of the wave propagating along its axis (guide wavelength).

## 2.3 Wave

A wave travelling in a transmission medium is for our purposes sufficiently characterized by

- its wavelength $\lambda$,
- its phase angle $\varphi(x)$ at a given position $x$, measured along the direction of propagation,
- the mean power $P$ it carries.

Mathematically, the wave is described by the *complex wave amplitude* (we shall further often call it simply *wave*)

$$a(x) = A\exp[j\varphi(x)] = \sqrt{2P}\exp[j\varphi(x)] \qquad (6)$$

The reason for using the square root of power as magnitude rather than the power itself is very essential: if two or more individual waves exist in a medium, they add up to form a resultant wave. To arrive at the resultant wave, we have to sum up *field strengths* (intensities) not powers of the individual waves. The wave magnitude must therefore be proportional to its field strength; and field strength is proportional to the square root of power[1]. In case of TEM waves, such as those existing in coaxial lines, voltages and currents are proportional to field strengths.

The factor 2 reflects the fact that $A$ is amplitude-type quantity (peak value) rather than effective value, so that

$$P = \frac{1}{2}|a(x)|^2 = \frac{1}{2}A^2 \qquad (7)$$

This power-intensity relation has a very practical consequence, sometimes overlooked in min-max power estimations of multiple-wave cases: To obtain resulting power, we must not sum powers of particular waves: we should sum square roots of powers, then take the square of the sum. The following example illustrates the difference.

**PROBLEM**: Two waves of the same type and frequency travel in the same direction; one carrying the power $P_1$=1000 W, the other $P_2$=500 W. Their mutual phase shift is unknown or may vary. What is the maximum and minimum power of the resulting wave?

**SOLUTION**: The maximum power case occurs when the two waves are in-phase. Then their amplitudes add algebraically. The minimum power case occurs when the two waves are phase-shifted by 180°. Then their amplitudes subtract algebraically. The corresponding powers are

$$P = \left(\sqrt{P_1} \pm \sqrt{P_2}\right)^2$$

yielding $P_{min}$ = 85.8 W, $P_{max}$ = 2914.2 W, ***not*** $P_{min}$ = 1000 – 500 = 500 W, $P_{max}$ = 1000 + 500 = 1500 W.

When a wave travels a distance $L$, its phase will decrease proportionally to the corresponding electrical distance $\theta$:

$$a(L) = a(0)\exp(-j2\pi L/\lambda) = a(0)\exp(-j\theta)$$

i.e.

$$\varphi(L) = \varphi(0) - j\theta$$

## 2.4 Signal source

In the theory of linear lumped-element circuits or circuits containing also TEM transmission lines (not waveguides), a signal source can be defined by two quantities:

- Open-circuit voltage amplitude (peak value) $V_S$
- Internal impedance $Z_S$

[1] More logically, the statement should be inverted: power is proportional to field strength amplitude squared.

Both quantities are in principle complex. If the studied system has only one signal source (which is our case), we can, without lack of generality, assume $V_S$ real.

A more general approach, applicable also to waveguides, is to characterize a signal source by another set of parameters:

- Available power $P_{av}$
- Source reflection coefficient $\Gamma_S = |\Gamma_S| \exp(j\varphi_S)$

### 2.4.1 Available Power

Available power is the maximum power a source is capable of delivering when varying its load impedance. This situation occurs when the load impedance[2] is equal to the complex conjugate of the source impedance. The same holds for reflection coefficients.

For sources in which voltages and currents can unambiguously be defined, the relation between the open-circuit voltage and the available power is

$$P_{av} = \frac{1}{8}\frac{V_S^2}{R_S} \qquad (8)$$

where $R_S$ is the real part of the source impedance $Z_S=R_S+jX_S$ (recall that $V_S$ is the *amplitude* of a harmonic signal, not its effective value; therefore the factor 1/8).

### 2.4.2 Source Reflection Coefficient

Similarly to the load, the source impedance and reflection coefficient are related by

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \qquad (9)$$

If the voltage $V_S$ could be reduced to zero without affecting anything else, the source would act simply as an impedance $Z_S$ (reflection coefficient $\Gamma_S$), similar to the load discussed above. Therefore, if a wave reflected from a load returns to the source, its power is partly absorbed in the source, partly re-reflected. The re-reflected wave contributes to the resultant wave travelling from the source towards load.

The source is said to be *matched* if $Z_S=Z_0$, or, equivalently, $\Gamma_S=0$. The source mismatch factor is defined by

$$m_S = 1 - |\Gamma_S|^2 \qquad (10)$$

A signal source may be a composite device, with a provision made to improve its impedance match. In particular, magnetrons are often cascaded with circulators. The wave reflected from the load is then absorbed not in the magnetron itself (which could damage it) but in the high-power termination connected to the third circulator port.

### 2.4.3 Power Delivered to Matched Load

The power which the source would deliver to a matched load (i.e. a load with zero reflection coefficient) can be obtained as the product of the source available power and the source mismatch factor:

$$P_{S0} = P_{av} m_S \qquad (11)$$

[2] Note that this is the impedance seen by the source at its terminals (reference plane), not the load impedance at the other end of the transmission medium, as defined in Section 2.1.

### 2.4.4 Total Power

The total power $P_{tot}$ delivered by the active portion of the source (ideal voltage source) is greater than the power delivered to the load. The reason is that part of this power is also dissipated in the real part of the source impedance. Note that in the case of maximum power transfer ($R_L$=$R_S$, $X_L$=-$X_S$) $P_{tot}$=2$P_{av}$, i.e. half of the total power is lost in the source itself. This is unacceptable in high-power sources (magnetron generators). The problem will be addressed in Section 5.

### 2.4.5 Magnetron

Unfortunately, the magnetron is not an easy signal source: its parameters (efficiency, internal impedance, generated frequency, etc.) are interrelated and depend on many factors, such as anode voltage and current, applied DC magnetic field intensity (electromagnet current), filament current, and even load impedance.

Nonetheless, for the purpose of the following explanations, we shall regard the magnetron a well-behaving, linear signal source. This approach is quite realistic if we consider the signal source to be a combination of a magnetron and a circulator and do not care for what happens inside. Problems of magnetrons will be discussed also in Section 5.

## 3. SYSTEM BEHAVIOR

This section describes the processes occuring in the waveguide. What we wish to arrive at and explain is the *steady-state* situation: in practical industrial systems, usually tens of nanoseconds are sufficient to reach steady state after an abrupt change. Circuit theory, theory of signal flow graphs, and scattering parameters formalism are general tools for obtaining the resulting formulas. However, for a better insight, it is very instructive to contemplate the onset of the wave propagation and the buildup of the resultant field in terms of successively arising particular waves.

To facilitate further discussion, the following conventions will be used:

- Amplitudes of waves travelling in the source-to-load direction (incident waves) will be denoted *b*.
- Amplitudes of waves travelling in the load-to-source direction (reflected waves) will be denoted *a*.
- Subscript S will indicate the position at the source-to-waveguide interface.
- Subscript L will indicate the position at the waveguide-to-load interface.
- Order *n* of a wave (the term will be explained below) will be specified by an additional subscript *n*.

Imagine the source is abruptly switched on. A consequence of that is the emergence of a wave travelling in the waveguide toward load. We shall call it *primary wave* or 1st-order incident wave. At the source output plane, the amplitude of this primary wave is $b_{S1}$. The power $P_1$ carried by this wave is the power which the source would deliver to a matched load:

$$P_1 = \frac{1}{2}|b_{S1}|^2 = P_{av} m_S$$

Hence (choosing $b_{S1}$ real, which we can do without loss of generality)

$$b_{S1} = \sqrt{2P_1} = \sqrt{2P_{av} m_S}$$

The primary wave propagates in the waveguide until, after travelling the electrical distance θ, reaches the load. The wave incident on the load is therefore

$$b_{L1} = b_{S1} \exp(-j\theta)$$

(recall that the transmission medium is assumed lossless). Now, depending on the load reflection coefficient, part of the wave penetrates to the load; the other part is reflected, giving rise to a wave travelling back toward source. We shall call it 1st-order reflected wave due to the fact that its *amplitude* is proportional to the 1st power of the load reflection coefficient[3]:

$$a_{L1} = b_{L1}\Gamma_L = b_{S1}\Gamma_L \exp(-j\theta)$$

The 1st-order reflected wave propagates in the waveguide until reaching the source, where

$$a_{S1} = a_{L1}\exp(-j\theta) = b_{S1}\Gamma_L \exp(-j2\theta)$$

There, depending on the source reflection coefficient, part of it penetrates back to the source; the other part is re-reflected, giving rise to another wave travelling toward load. We shall call it 2nd-order incident wave; its amplitude at the source and the load is, respectively

$$b_{S2} = a_{S1}\Gamma_S = b_{S1}\Gamma_S\Gamma_L \exp(-j2\theta)$$

$$b_{L2} = b_{S2}\exp(-j\theta) = b_{S1}\Gamma_S\Gamma_L \exp(-j3\theta)$$

The 2nd-order incident wave behaves exactly as the primary wave, giving rise to 2nd-order reflected wave, which in turn gives rise to 3rd-order incident wave. This process continues theoretically infinitely (taking theoretically infinite time), then the steady state is reached. Practically, only a finite number of higher-order contributions is significant and a practically acceptable steady state occurs after a finite time (those already mentioned tens of ns).

Generally, nth-order incident and reflected waves at the load plane are

$$b_{Ln} = b_{S1}\Gamma_S^{n-1}\Gamma_L^{n-1}\exp[-j\theta(2n-1)] \quad \text{n=1, 2, 3, …} \tag{12}$$

$$a_{Ln} = b_{Ln}\Gamma_L = b_{S1}\Gamma_S^{n-1}\Gamma_L^{n}\exp[-j\theta(2n-1)] \quad \text{n=1, 2, 3, …} \tag{13}$$

The steady state situation is illustrated in Fig. 3 and can be described as follows:

1. The resultant wave propagating toward load (incident wave) is composed of the primary wave and an infinite number of higher-order contributions with decreasing amplitudes. Their origin is multiple reflections between source and load. At the load plane, the incident wave is

$$b_L = \sum_{n=1}^{\infty} b_{Ln} = b_{S1}\sum_{n=1}^{\infty}\Gamma_S^{n-1}\Gamma_L^{n-1}\exp[-j\theta(2n-1)] \tag{14}$$

2. The resultant wave propagating back toward source (reflected wave) is again composed of an infinite number of contributions with decreasing amplitudes. At the load plane, the reflected wave is

$$a_L = \sum_{n=1}^{\infty} a_{Ln} = \Gamma_L\sum_{n=1}^{\infty} b_{Ln} = \Gamma_L b_L$$

   The complex amplitude ratio of the resultant reflected wave and the resultant incident wave is equal to the load reflection coefficient, as is the ratio of each particular pair of contributions with a given order.

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[3] Similarly, a reflected wave will be said to be of nth order if its amplitude is proportional to nth power of *load* reflection coefficient, $(\Gamma_L)^n$. An incident wave will be said to be of nth order if it gives rise to nth-order reflected wave. So, each incident and reflected wave of the same order are coupled by the load reflection coefficient.

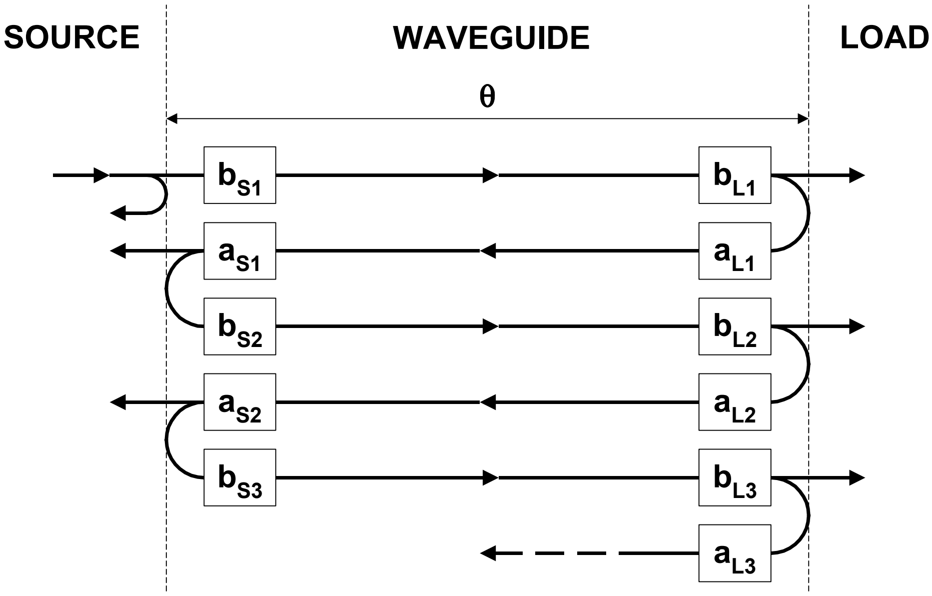


Fig. 3: Multiple reflections in waveguide

The infinite series can in fact be easily summed, realizing that it is a geometric series with the quotient $q = \Gamma_S \Gamma_L \exp(-j2\theta)$ as we shall prove now. Indeed, after the substitution $k$=$n$-1, which also implies 2$n$-1=2$k$+1, the formula for $b_L$ becomes

$$b_L = b_{S1} \sum_{k=0}^{\infty} \Gamma_S^k \Gamma_L^k \exp(-j\theta) \exp(-j2\theta k) = b_{S1} \exp(-j\theta) \sum_{k=0}^{\infty} \left[\Gamma_S \Gamma_L \exp(-j2\theta)\right]^k$$

Recalling that, for |q|<1,

$$\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$$

one obtains

$$b_L = b_{S1} \frac{\exp(-j\theta)}{1 - \Gamma_S \Gamma_L \exp(-j2\theta)}$$

Expressing $b_{S1}$ in terms of source available power, the wave incident on the load is

$$b_L = \sqrt{2P_{av} m_S} \frac{\exp(-j\theta)}{1 - \Gamma_S \Gamma_L \exp(-j2\theta)} \qquad (15)$$

and the wave reflected from the load is

$$a_L = \Gamma_L b_L \qquad (16)$$

which are the final formulas we wanted to arrive at.

# 4. POWER TRANSFER IN THE SYSTEM

This section discusses some important points concerning powers and power transmission in the system.

Power carried by the wave $b_L$ (incident power) is

$$P_i = \frac{1}{2} |b_L|^2 = P_{av} \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_S \Gamma_L \exp(-j2\theta)|^2} \qquad (17)$$

This power can be measured by a directional coupler connected in the waveguide such as to sample the forward wave.

Power carried by the wave $a_L$ (reflected power) is

$$P_r = \frac{1}{2}|a_L|^2 = P_i|\Gamma_L|^2$$

This power can be sampled by a directional coupler oriented in the reverse direction.

Next, we shall discuss some implications of the derived formulas.

### 4.1 Power Absorbed in the Load

Power absorbed in load is of prime concern in the high-power applications. Following the energy conservation principle, the net power absorbed in the load is the difference between the incident power and the reflected power:

$$P_L = P_i - P_r = P_i\left(1-|\Gamma_L|^2\right) = P_{av}\frac{\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\Gamma_L\exp(-j2\theta)|^2} \tag{18}$$

Equation (18) can be rewritten as

$$P_L = P_{av}g$$

where

$$g = \frac{\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\Gamma_L\exp(-j2\theta)|^2} \tag{19}$$

is the power transmission coefficient also known as *transducer gain ratio*. It is usually expressed in dB as

$$G = 10\log(g)$$

As seen, the transducer gain[4] relates the power actually absorbed in the load to the available source power. It is therefore a quantity of basic interest.

Transducer gain expressed in terms of reflection coefficients has a very illustrative interpretation. We can write it as a product of three factors:

$$g = m_S\, d_{SL}\, m_L \tag{20}$$

where

$$m_S = 1-|\Gamma_S|^2$$

$$m_L = 1-|\Gamma_L|^2$$

$$d_{SL} = \frac{1}{|1-\Gamma_S\Gamma_L\exp(-j2\theta)|^2} \tag{21}$$

Two of them ($m_S$, $m_L$) are the familiar mismatch factors; the third ($d_{SL}$) we shall call the *interference factor*.

The source mismatch factor defines what fraction of the available power would be transferred to a matched load: indeed, if $\Gamma_L$=0, then $g$=$m_S$. We have stated that already in Equation (11).

[4] In lossy systems, perhaps the more logical terms are transducer *loss* ratio ($t$) and transducer loss expressed in dB ($T$). These are related to transducer gain as $t = 1/g$, $T = - G$.

The load mismatch factor defines what fraction of the power available from a *matched*[5] source would be transferred to a given load: indeed, if $\Gamma_S=0$, then $g=m_L$.

It seems at the first sight that when both the source and the load are mismatched, the transducer gain will simply be the product of the two mismatch factors: a fraction $m_S$ of the available power couples to the transmission line, out of which only a fraction $m_L$ couples to the load. But this is not the case: if there are mismatches at *both ends*, multiple reflections occur between source and load as elaborated above, modifying the overall power transmission. Exactly this is the effect expressed by the interference factor.

Both the energy conservation principle and the analysis of (19) imply that for passive loads $g \le 1$, that is, power absorbed in a load can never be greater than the power available from the source. On the contrary to this, the power of the incident wave can be substantially higher, as will be shown later.

Equation (21) reveals that the interference factor depends not only on the magnitudes of the source and load reflection coefficients but also on their phases, and on the electrical length θ of the interconnecting wavegide. This is due to the term

$$h = \Gamma_S \Gamma_L \exp(-j2\theta) = |\Gamma_S \Gamma_L| \exp[j(\varphi_S + \varphi_L - 2\theta)] = |\Gamma_S \Gamma_L| \exp(j\varphi_h)$$

appearing in the denominator. In the practice of industrial and many other applications, the phase angles $\varphi_S$, $\varphi_L$, θ are unknown and, more than that, cannot at all be assumed constant. It is therefore important and only possible to know the limits of the interference factor and derived quantities (like transducer gain, power absorbed in load, incident power) for arbitrary phase angles. A minimum of interference factor occurs when the denominator is highest, and vice versa. The denominator is highest when the phasing is such that $h$ is real and positive: $h = |\Gamma_S \Gamma_L|$. The denominator is smallest when the term is real and negative: $h = -|\Gamma_S \Gamma_L|$. Consequently,

$$d_{SL\min} = \frac{1}{(1+|\Gamma_S\Gamma_L|)^2}, \qquad d_{SL\max} = \frac{1}{(1-|\Gamma_S\Gamma_L|)^2}$$

It is apparent that:

1. When at least one end is matched, the interference factor is equal to unity.
2. The worse the mismatches (i.e. the higher the $|\Gamma_S \Gamma_L|$ product), the more the extremes of interference factor deviate from unity. In the limit $|\Gamma_S \Gamma_L| \to 1$, the minimum is 0.25 and the maximum tends to infinity.

The limits of transducer gain are

$$g_{\min,\max} = \frac{\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{(1\pm|\Gamma_S\Gamma_L|)^2}$$

The numerator assures that transducer gain cannot exceed unity, hence conforms with the energy conservation principle. The following can also be stated:

1. If both ends are matched, then $g = 1$ and the whole available power is transmitted to load.
2. If one end is matched, transducer gain is equal to the mismatch factor of the other end, independent of phases.
3. If the reflection coefficient *magnitudes* of both ends are equal, then $g_{\max} = 1$ (it means that by proper phasing the impedance conjugation condition for the maximum power transmission can be achieved).
4. If the reflection coefficient magnitudes of both ends are inequal, then $g_{\max} < 1$ (complete power transfer by mere changing phases or waveguide length cannot be achieved).

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[5] It is to be stressed that the *match* is related to the reference impedance $Z_0$, i.e. to the characteristic impedance of the interconnecting transmission line or waveguide.

## *4.2 Incident Power*

Equation (17) shows that the incident power formula also comprises the interference factor:

$$P_i = P_{av} \frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_L \exp(-j2\theta)|^2} = P_{av} m_S d_{SL}$$

Therefore, like absorbed power, incident power depends on the phases of the source and load reflection coefficients as well as electrical length θ of the interconnecting wavegide. However, due to the missing load mismatch factor, this dependence is completely different. In particular, the incident power can be much greater than the power available from the source, as will be shown next. Physically, the effect is a result of the vector summation of partial waves originating from multiple reflections of the primary wave, as explained in Section 3. Since the contributions add vectorially, the incident wave magnitude, depending on their phasing, may be greater than the original wave. However, the higher the incident power, the higher also the reflected power: their difference (power absorbed in load) can never exceed the source available power.

The extremes of the incident power are

$$P_{i\,\min,\max} = P_{av} \frac{1-|\Gamma_S|^2}{(1\pm|\Gamma_S\Gamma_L|)^2}$$

Let us analyze $P_{\mathrm{i\,max}}$ to see if it can be greater than $P_{\mathrm{av}}$. A simple manipulation of the relation $1-|\Gamma_S|^2 > (1-|\Gamma_S||\Gamma_L|)^2$ shows that this case actually occurs when

$$|\Gamma_S| < \frac{2|\Gamma_L|}{1+|\Gamma_L|^2}$$

or, equivalently,

$$|\Gamma_L| > \frac{1}{|\Gamma_S|}\left(1-\sqrt{1-|\Gamma_S|^2}\right)$$

If, for example, the source VSWR=3, i.e. $|\Gamma_S|=0.5$, the incident power is greater than the available power for all loads with $|\Gamma_L|>0.27$ i.e. VSWR>1.73. The worse the both mismatches, the higher the maximum incident power. **Table 1**, assuming both reflection coefficients equal, helps get a feeling for the relations. Miscelaneous computations of similar kind can be performed using the PowTrans program.

Table 1: Minimum and maximum incident power for equally mismatched source and load

| $\|\Gamma_S\|=\|\Gamma_L\|$ | VSWR | RL (dB) | $P_{min}/P_{av}$ | $P_{max}/P_{av}$ | $P_{min}/P_{av}$ (dB) | $P_{max}/P_{av}$ (dB) |
|---|---|---|---|---|---|---|
| 0.1 | 1.22 | 20.00 | 0.97 | 1.01 | -0.13 | 0.04 |
| 0.2 | 1.50 | 13.98 | 0.89 | 1.04 | -0.52 | 0.18 |
| 0.3 | 1.86 | 10.46 | 0.77 | 1.10 | -1.16 | 0.41 |
| 0.4 | 2.33 | 7.96 | 0.62 | 1.19 | -2.05 | 0.76 |
| 0.5 | 3.00 | 6.02 | 0.48 | 1.33 | -3.19 | 1.25 |
| 0.6 | 4.00 | 4.44 | 0.35 | 1.56 | -4.61 | 1.94 |
| 0.7 | 5.67 | 3.10 | 0.23 | 1.96 | -6.39 | 2.92 |
| 0.8 | 9.00 | 1.94 | 0.13 | 2.78 | -8.73 | 4.44 |
| 0.9 | 19.00 | 0.92 | 0.06 | 5.26 | -12.37 | 7.21 |
| 0.95 | 39.00 | 0.45 | 0.03 | 10.26 | -15.70 | 10.11 |
| 0.99 | 199.00 | 0.09 | 0.01 | 50.25 | -22.95 | 17.01 |

The following conclusions can be drawn concerning the incident power:

1. For a given source and load mismatch, the power carried by the wave travelling in the waveguide toward source (incident power) can be lower or higher, depending on phases of both reflection coefficients and waveguide length. For gross mismatches, it can be even substantially higher than the source available power.
2. Therefore, the incident power is *not* a measure of the power absorbed in the load, which is the actual concern in high-power applications. Relying solely on incident power measurement (i.e. using a single directional coupler) may lead to misinterpretation; especially when neither source nor load are well matched and do not have constant reflection coefficient (which is typical for magnetron installations without circulators). To properly measure power absorbed in a load, two directional couplers are required: one sampling incident power, the other sampling reflected power.
3. The importance of incident power itself lies in the fact that it is a measure of the *field strength* inside the waveguide. For gross mismatches at both ends, the field inside the waveguide may be very strong despite the low net transmitted power. In fact, such arrangenment closely resembles a resonator with all the consequences (e.g a dip in the transmitted power accompanied by an excessive loss in the real transmission medium). Objects, like tuning stubs, located in such environment may experience overheating; theoretically, even electric breakdown may occur.
4. The situation when the incident power is greater than the source available power does not violate the energy conservation law because an adequate part of this power is reflected back to signal source. In final effect (assuming no transmission losses), the source delivers exactly the power absorbed in load[6].

Note: Rather than evaluating the comparatively complex formulas derived, useful quick estimates can be obtained taking into account only the primary wave, 1st-order reflected wave, and 2nd-order incident wave. The better the source and load match, the more accurate this approach. For VSWR≤2, the absorbed power error is below ±10 %. Remember that to obtain the limits of the total power of the incident wave, you have to proceed as in the Problem in Section 2.3.

# 5. MAGNETRON AS A SIGNAL SOURCE

As noted in Section 2.4, the total power delivered by the active portion of a generator (i.e. ideal voltage source) is always greater than the power delivered to the load. Namely, part of the total power is dissipated in the source impedance. This has an important consequence on the operation of high-power microwave generators. The problem is discussed in the present section. The relations are best explained in terms of circuit parameters (voltages, currents, impedances).

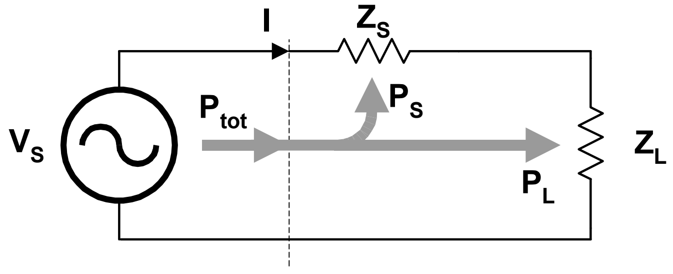


Fig. 4: Illustration to power definitions

[6] For a magnetron with a circulator, the source in this sense means the combination of the two. Looking inside, however, the magnetron sees a nearly perfect match and delivers the corresponding power (see Section 2.4.3) regardless of the load impedance. The whole power reflected from the load is absorbed in the circulator auxiliary load.

The situation is illustrated in Fig. 4, where

$$P_L = P_{av} \frac{4R_S R_L}{\left|Z_S + Z_L\right|^2}$$

is the power absorbed in the load[7], hence the transducer gain is

$$g = \frac{4R_S R_L}{\left|Z_S + Z_L\right|^2}$$

Power dissipated in the source impedance is

$$P_S = P_{av} \frac{4R_S^2}{\left|Z_S + Z_L\right|^2}$$

and the source total power is

$$P_{tot} = \frac{1}{2}\mathrm{Re}\left\{V_S I^*\right\} = P_S + P_L = P_{av} \frac{4R_S\left(R_S + R_L\right)}{\left|Z_S + Z_L\right|^2}$$

(the asterisk denotes complex conjugation). The problem of high-power microwave generators can be described as follows: If the source and load are matched ($R_L$=$R_S$, $X_L$=-$X_S$), the source delivers its available power $P_{av}$ to the load. On the other hand, the same amount of power is lost in the source internal impedance itself (in its real part $R_S$). The active device of the source must therefore supply twice the available power ($P_{tot}$=2 $P_{av}$), hence the source efficiency cannot theoretically exceed 50%. This is unacceptable in high-power generation. In order that the efficiency be higher, the power lost in $R_S$ must be much lower. This is possible only when $R_S$<<$R_L$, i.e. the magnetron must act as a strongly mismatched signal source.

The conclusion is that the source match is not important in high-power microwave generators: the prime concern is the efficiency with which the DC input power is converted to microwave power.

As a result of the excessive source mismatch, the available power turns out to be much higher than the power delivered to a load. The term available power therefore loses its real meaning and becomes only a fictitious quantity: an extrapolation, useful for mathematical analysis.

Fig. 5 illustrates the situation, considering for simplicity both source and load impedances real ($X_L$=0, $X_S$=0). The figure shows the total power, load power, and the power lost in source as functions of load impedance. It also shows the fraction $\eta_D$ of the total power that is transferred to the load:

$$\eta_D = \frac{P_L}{P_{tot}} = \frac{R_L}{R_S + R_L} \qquad (22)$$

The quantity can be described as *power distribution efficiency*.

When $R_L$ is zero, the total power is four times the available power and, logically, all this power is lost in the source. When increasing the load impedance, the total power decreases. The same holds for the power lost in the source, however this decreases faster because part of the total power now goes to the load. The load power grows until $R_L$=$R_S$. When $R_L$=$R_S$, the familiar impedance matching condition occurs: the total power is twice the available power, and is equally distributed between the source and load impedances. The efficiency $\eta_D$ = 50% in this situation. When increasing the load impedance above this point, all three powers decrease. However, the difference between the total power and the load power is ever smaller so the efficiency increases. This confirms the qualitative conclusion stated above that the source impedance must be low relative to the load impedance to achieve high efficiency.

[7] This is a circuit-formalism equivalent of Equation (18); however, $Z_L$ now means the load impedance transformed by the waveguide to magnetron reference plane. $Z_L$ therefore incorporates the interference factor.

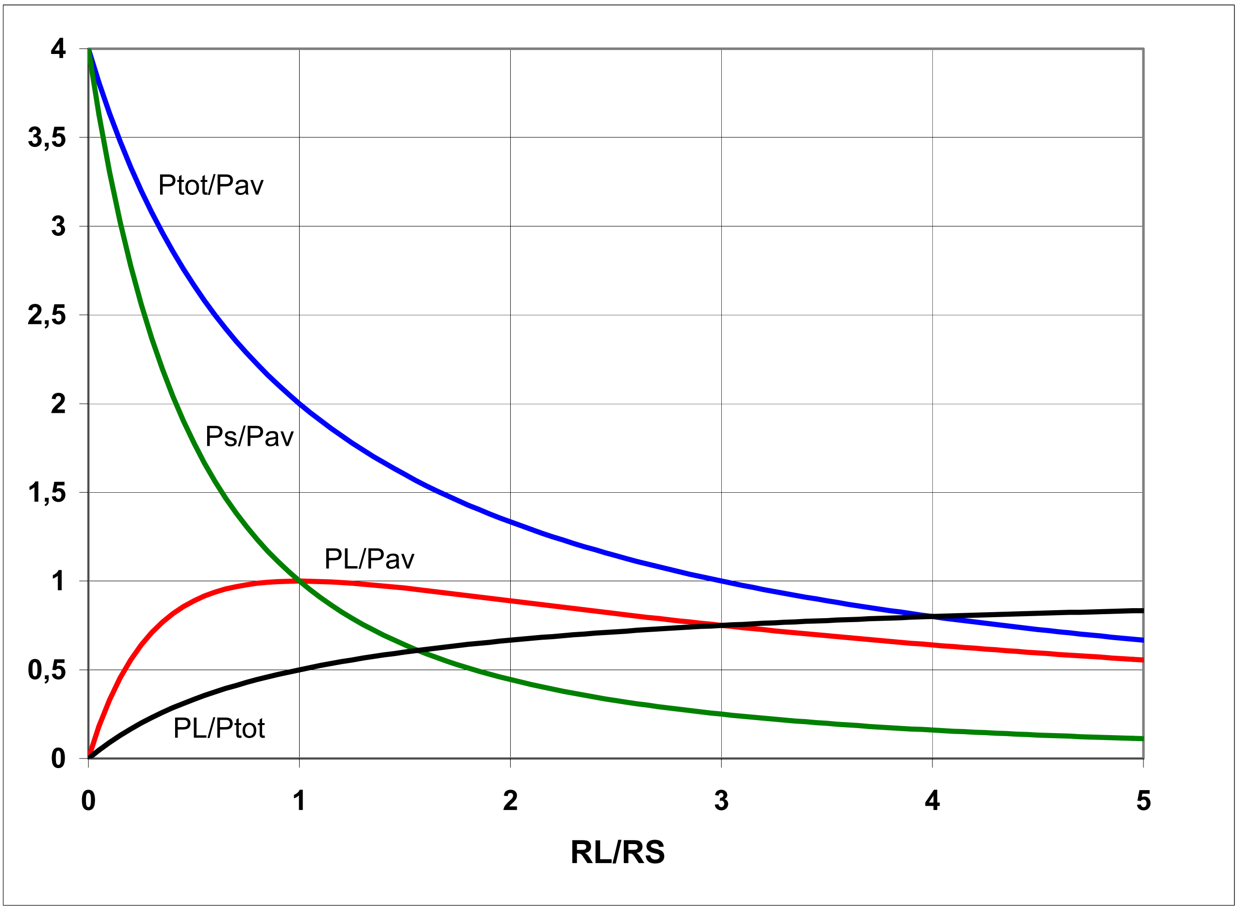


Fig. 5: Source total power relative to the available power ($P_{tot}/P_{av}$) and its distribution to load and source internal impedance for varying $R_L/R_S$ ratio. The curve $P_L/P_{tot}$ is the distribution efficiency $\eta_D$.

Now, a practical question arises: When the load is matched ($Z_L$=$Z_0$), what must the source reflection coefficient be in order that the magetron overall efficiency $\eta$ have a desired value?

The overall efficiency can be defined as power absorbed in a matched load divided by the input DC power. Then

$$\eta = \frac{P_L}{P_{DC}} = \frac{P_{tot}}{P_{DC}} \frac{P_L}{P_{tot}} = \eta_C \eta_D$$

where $P_{DC}$ is the input DC power and $\eta_C$ is the efficiency of DC-to-$P_{tot}$ conversion. Such defined efficienciess are not load impedance-dependent ($Z_L$ has been made equal to $Z_0$), can therefore serve as meaningful magnetron specifications. From the knowledge or an estimate of $\eta_C$, the value of $\eta_D$ can be obtained. Then, using (22), the source impedance is

$$R_S = Z_0 \frac{1-\eta_D}{\eta_D}$$

Converting $R_S$ to the more general reflection coefficient, we obtain

$$|\Gamma_S| = 1 - 2\eta_D$$

The case is best illustrated in the following example.

**PROBLEM**: A magnetron's anode voltage is $V_a$ = 18 kV, anode current is $I_a$ = 5 A, and its overall efficiency should be $\eta$ = 85%. What must be magnetron's internal impedance expressed in terms of

VSWR? What is its available power in such case? Assume the conversion and power distribution efficiencies equal.

**SOLUTION**: The power distribution eficiency and the conversion efficiency are

$$\eta_D = \eta_C = \sqrt{\eta} = \sqrt{0.85} = 0.922$$

Using this,

$$|\Gamma_S| = |1 - 2\times 0.922| = 0.844$$

$$VSWR = (1+|\Gamma_S|)/(1-|\Gamma_S|) = 11.8$$

which is a severe mismatch indeed. The input DC power is

$$P_{DC} = V_a I_a = 90\,kW$$

The total source power is

$$P_{tot} = P_{DC}\eta_C = 83\,kW$$

The power absorbed in a matched load is

$$P_L = P_{DC}\eta = 76.5\,kW$$

The available power can be computed from Equation (11) as

$$P_{av} = P_L \Big/ \left(1-|\Gamma_S|^2\right) = 3.47\,P_L = 7265.8\,kW$$

This is indeed a fictitious value. Nevertheless, it can be used in system calculations to arrive at realistic final results.

Distribution of powers in the system is illustratively depicted in Fig. 6.

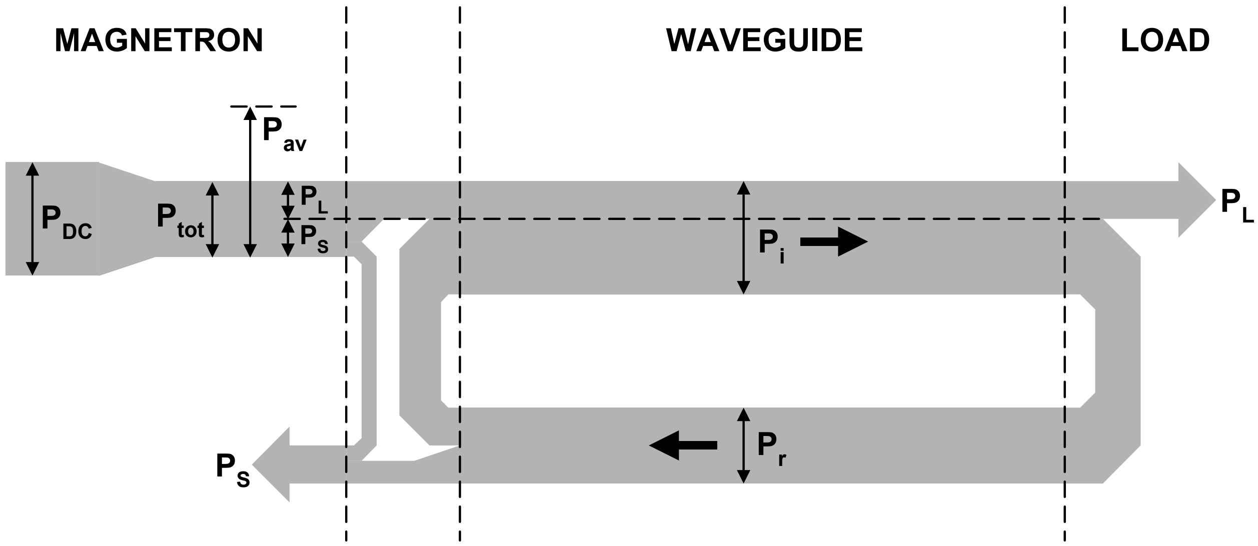


Fig. 6: Power distribution in the transmission system with a magnetron

A magnetron as a grossly mismatched source is, however, a nuisance because power transmission in such a system with is very sensitive to the phase of load reflection coefficient and the waveguide length. Also, when the condition of maximum power transfer is approached, magnetron behavior is unpredictable and the magnetron can even be damaged. To avoid these effects, a circulator can be placed between the magnetron and the load. With an ideal circulator, this has the following consequences:

- The magnetron always looks into a matched load.
- Looking from outside, the magnetron-circulator cascade also appears matched.

- The power of the wave incident on the load is always equal to the power which would be delivered to a matched load. (It does not vary with load variations.)
- The reflected power is fully absorbed in the circulator (diverted to the auxiliary load).

Modification of Fig. 6 for the case of a circulator is left to the reader.

# 6. WHAT SHOULD BE MEASURED AND HOW

Based on the results obtained in the previous sections, this section suggests what quantities should be actually measured to the satisfaction of a high-power microwave system user, how the measurements can be performed and what are the measurement worst-case accuracy limitations.

The basic quantities of interest in high-power microwave applications are

- Power $P_L$ absorbed in the load
- Magnitude $|\Gamma_L|$ of the load reflection coefficient

The power absorbed in the load is interesting for an obvious reason of using energy as efficiently as possible as well as for us being able to relate the processes in the working space to the delivered power.

The load reflection coefficient is important to be monitored among others in order that the magnetron load impedance be kept within recommended or permitted limits. In other words, magnetrons must be protected against reflected waves (simpler applications do not use circulators) because these could impair their proper operation or cause damage.

Both quantities of interest can be obtained from the measurement of other two quantities:

- Power $P_i$ of the wave incident on the load
- Power $P_r$ of the wave reflected from the load

Then the power absorbed in the load is

$$P_L = P_i - P_r \tag{23}$$

and the magnitude of the load reflection coefficient is

$$|\Gamma_L| = \sqrt{P_r / P_i} \tag{24}$$

$P_i$ and $P_r$ can be obtained by sampling the incident and reflected wave by means of directional couplers.

As already noted, it is not uncommon that only one directional coupler is used, sampling the incident wave. The coupled power is supposed to be proportional to the power absorbed in the load. This can, however, be true only in two special cases:

- When the load is matched
- When reflection coefficients of both load and source are constant (including their phase angles).

Neither of these conditions is met in practice. Then a paradoxical situation may occur that the load power "measured" in such a way is greater then the DC input to magnetron. This can happen with poorly matched magnetrons (operating without circulators) and poorly matched loads. The correct approach is therefore to measure both incident and reflected powers and use their difference and ratio to arrive at power absorbed in load and load reflection coefficient[8].

[8] It is worth noting that there exist other principles which enable to measure both quantities of interest. One of them is the six-port reflectometer principle. The six-port reflectometer can measure both complex reflection coefficient and incident power. Moreover, the knowledge of *complex* reflection coefficient makes it possible to correct for the systematic errors which severly limit the measurement accuracy when using only scalar (power) data provided by couplers. Also, the knowledge of complex reflection coefficient is a key to effective automatic impedance matching algorithms.

## 6.1 Directional Couplers

In this section, expressions for coupled waves of directional couplers used for sampling incident and reflected waves are declared as a starting point for further analysis.

Directional couplers for high-power waveguide applications have very weak coupling (below –40 dB), so they do not introduce any significant reflection in the main guide and can be regarded perfectly matched. Therefore, for the purpose of the present analysis, a directional coupler will be characterized by the following two, generally complex, quantities:

Coupling factor $c$

Directivity $d$

Note that they are related with the commonly given dB-values by

$$C = 20\log|c|$$

$$D = -20\log|d|$$

A coupler connected so as to sample the wave $b_L$ incident on the load will be called a *forward coupler*. Its coupling factor and directivity will be denoted $c_i$, $d_i$, respectively.

A coupler connected so as to sample the wave $a_L$ reflected from the load will be called a *reverse coupler*. Its coupling factor and directivity will be denoted $c_r$, $d_r$, respectively.

The wave emerging from the coupled port of the forward coupler is then

$$b_{ci} = b_L c_i + a_L c_i d_i = b_L c_i (1 + \Gamma_L d_i)$$

The wave emerging from the coupled port of the reverse coupler is

$$b_{cr} = a_L c_r + b_L c_r d_r = b_L c_r (\Gamma_L + d_r)$$

These two equations enable us to assess errors when measuring the powers and the magnitude $|\Gamma_L|$ of load reflection coefficient $\Gamma_L = a_L / b_L$.

For a case of ideally directive couplers ($d_i$=0, $d_r$=0), the equations reduce to

$$b_{ci} = b_L c_i$$

$$b_{cr} = b_L c_r \Gamma_L$$

Apart from finite directivities, another source of measurement error is inaccurate knowledge of the coupling factors. To account for this, the terms *nominal* (supposed) coupling factors $c_{in}$, $c_{rn}$ will be used as opposed to the *actual* values $c_i$, $c_r$.

## 6.2 Power Measurement Using Directional Couplers

Power carried by the wave emerging from the coupled port of a coupler is

$$P_c = \frac{1}{2}|b_c|^2$$

Applying this to the samples of the incident ($b_{ci}$) and reflected ($b_{cr}$) wave, we obtain

$$P_{ci} = P_i |c_i|^2 |1 + \Gamma_L d_i|^2$$

$$P_{cr} = P_i |c_r|^2 |\Gamma_L + d_r|^2$$

where

$$P_i = \frac{1}{2}|b_L|^2$$

is the power of the wave incident on the load, the reflected power being $P_r = P_i|\Gamma_L|^2$.

Powers $P_{ci}$ and $P_{cr}$ can be measured by power meters connected to the coupled ports. For the purpose of this analysis, we shall assume that these powers are measured accurately.

Obviously, with perfect directional couplers ($d_i$=0, $d_r$=0), $P_{ci}$ is proportional to the incident power $P_i$ and $P_{cr}$ is proportional to the reflected power:

$$P_{ci} = |c_i|^2 P_i$$

$$P_{cr} = |c_r|^2 P_i|\Gamma_L|^2 = |c_r|^2 P_r$$

Therefore, to arrive at $P_i$ and $P_r$, the coupled powers $P_{ci}$ and $P_{cr}$ are to be divided by $|c_i|^2$ and $|c_r|^2$, respectively. However, since the true values of the coupling factors are not exactly known; we can only use some supposed (nominal) values $c_{in}$, $c_{rn}$ to carry out the division. What we obtain are *measured* values of the incident and reflected power

$$P_{mi} = \frac{P_{ci}}{|c_{in}|^2} = P_i\left|\frac{c_i}{c_{in}}\right|^2 |1+\Gamma_L d_i|^2 \qquad (25)$$

$$P_{mr} = \frac{P_{cr}}{|c_{rn}|^2} = P_i\left|\frac{c_r}{c_{rn}}\right|^2 |\Gamma_L + d_r|^2 \qquad (26)$$

As we see, the measured values $P_{mi}$ and $P_{mr}$ in general differ from the actual powers $P_i$ and $P_r$. As already stated in Section 6.1, the two main sources of the measurement errors are

1. Nonzero directivity of the couplers
2. Not exactly known coupling factors of the couplers

Both these errors will now be investigated.

### 6.2.1 Errors Due to Finite Directivity

Suppose for instant that the coupling factors are exactly known, i.e. $c_{in}=c_i$, $c_{rn}=c_r$. Equations (25), (26) then become

$$P_{mi} = P_i|1+\Gamma_L d_i|^2 \qquad (27)$$

$$P_{mr} = P_i|\Gamma_L + d_r|^2 \qquad (28)$$

For a given load reflection coefficient magnitude, the measurement results depend on the phase of $\Gamma_L$. Because we are not able to measure phase, nor the phases of the directivity factors are known, the power measurement uncertainties can only be expressed in terms of the minimum and maximum values for all possible phases of $\Gamma_L$, $d_i$, and $d_r$.

We have to distinguish between two cases:

- Case Known: $|d|$ is a *known* directivity (that is to say, for instance, that the coupler's directivity *is* 25 dB).
- Case Max: A *maximum* guaranteed value $d_{max}$ of the directivity is defined (that is to say, for instance, that the coupler's directivity is *higher than* 25 dB). In that case $|d|$ is unknown and can lie anywhere in the interval $0 \le |d| \le d_{max}$.

#### *6.2.1.1 Limits of Incident Power*

The situation for $P_{mi}$ is rather straightforward due to the fact that $|\Gamma_L d_i| < 1$. The situation is illustrated in Fig. 7, showing possible values of the vector $A = 1 + \Gamma_L d_i$. For Case Known, the limits of $P_{mi}$ are

$$P_{mi\,\min} = P_i\left(1 - |\Gamma_L|\,|d_i|\right)^2$$

$$P_{mi\,\max} = P_i\left(1 + |\Gamma_L|\,|d_i|\right)^2$$

For Case Max , the limits are

$$P_{mi\,\min} = P_i\left(1 - |\Gamma_L|\, d_{i\,\max}\right)^2$$

$$P_{mi\,\max} = P_i\left(1 + |\Gamma_L|\, d_{i\,\max}\right)^2$$

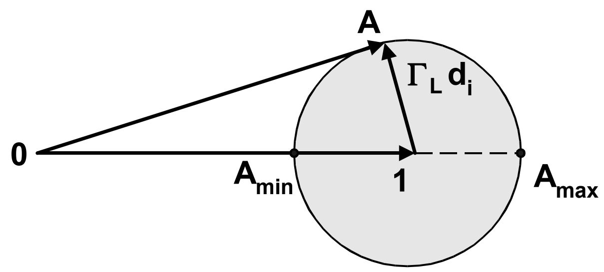


Fig. 7: Illustration to obtaining the limits of incident power

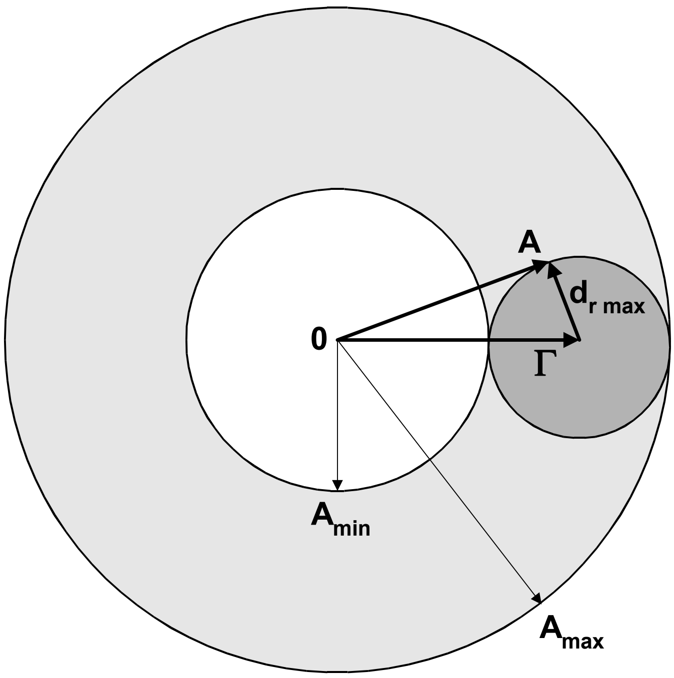


Fig. 8: Illustration to obtaining the limits of reflected power: case of reflection coefficient modulus greater than dr max. In the opposite case the darker circle always contains zero, hence Amin=0

#### *6.2.1.2 Limits of Reflected Power*

For Case Known, the limits of $P_{\mathrm{mr}}$ are obtained exactly as the limits of $P_{\mathrm{mi}}$:

$$P_{mr\,\min} = P_i\left(|\Gamma_L| - |d_r|\right)^2$$

$$P_{mr\max} = P_i\left(|\Gamma_L| + |d_r|\right)^2$$

Note that if the modulus of reflection coefficient equals that of $d_r$, a case may occur, depending on their phasing, when the measured reflection coefficient will be zero.

For Case Max, we must treat separately the instances $|\Gamma_L| > d_{r\max}$ and $|\Gamma_L| \le d_{r\max}$. The situation is illustrated in Fig. 8, showing possible values of the vector $A = \Gamma_L + d_r$ for all phases of both $\Gamma_L$ and $d_r$. The darker circle represents the area of all possible values of $A$ when the phase of $d_r$ is arbitrary and its magnitude varies between 0 and $d_{r\max}$. The lighter-shaded area represents the case when also phase of $\Gamma_L$ varies. The figure shows the case $|\Gamma_L| > d_{r\max}$. In both cases the maximal measured reflected power is

$$P_{mr\max} = P_i\left(|\Gamma_L| + d_{r\max}\right)^2$$

If $|\Gamma_L| > d_{r\max}$, the minimal measured power is

$$P_{mr\min} = P_i\left(|\Gamma_L| - d_{r\max}\right)^2$$

However, if $|\Gamma_L| \le d_{r\max}$, the minimal power can be zero:

$$P_{mr\min} = 0$$

This is because in this case the darker circle in Fig. 8 always contains zero.

### 6.2.2 Errors Due to Unknown Coupling

Suppose for instant that the directivity of the couplers is ideal, i.e. $d_i$=0, $d_r$=0. The formulas (24), (25) then become

$$P_{mi} = P_i\left|\frac{c_i}{c_{in}}\right|^2, \quad P_{mr} = P_r\left|\frac{c_r}{c_{rn}}\right|^2 \qquad (29)$$

and the measurement error is caused by the inaccurate knowledge of the coupling factors $c_i$, $c_r$.

It is a commom practice that the coupling factor uncertainty, either specified by the manufacturer or obtained by any calibration, is expressed in decibels as $\Delta C \ge 0$. The actual coupling $C$ (dB) is then limited by

$$C_{\min} = C_n - \Delta C\,, \quad C_{\max} = C_n + \Delta C$$

where $C_n = 20\log|c_n|$ is the nominal (supposed) coupling expressed in dB. If the measured powers are also expressed in dB units (e.g. dBm), then $\Delta C$ is the power measurement uncertainty, too.

In linear scale, we define the limits (real numbers)

$$c_{\min} = |c_n|10^{-\Delta C/20}\,, \quad c_{\max} = |c_n|10^{+\Delta C/20}$$

or

$$\left|\frac{c_{\min}}{c_n}\right|^2 = 10^{-\Delta C/10}\,, \quad \left|\frac{c_{\max}}{c_n}\right|^2 = 10^{+\Delta C/10}$$

Consequently, using (28) and separating the formulas for the forward and the reverse coupler, we have

$$P_{mi\min} = P_i\,10^{-\Delta C_i/10}$$

$$P_{mi\max} = P_i\,10^{\Delta C_i/10}$$

$$P_{mr\min} = P_r\,10^{-\Delta C_r/10}$$

$$P_{mr\max} = P_r\,10^{\Delta C_r/10}$$

Typical values of the coupling uncertainty-introduced error factor are summed up in **Table 2**. If, for example, the coupling factor is defined with the uncertainty of ±0.5 dB, which is quite a good accuracy, the actual power of 10 kW can be measured anywhere between 8913 W and 11220 W.

Table 2: Power measurement error caused by inaccurately known coupling factor

| $\Delta C$ (dB) | 0 | 0.1 | 0.2 | 0.5 | 1 | 2 |
|---|---|---|---|---|---|---|
| $10^{\Delta C/10}$ | 1 | 1.023 | 1.047 | 1.122 | 1.259 | 1.585 |

### 6.2.3 Total Error

The aggregate power measurement uncertainty formulas, including both directivity and coupling factor inaccuracy errors, are obtained by combination of the two effects. For Case Known, the total uncertainties are

$$P_{mi\,\min} = P_i\left(1-|\Gamma_L|\,|d_i|\right)^2 10^{-\Delta C_i/10}$$

$$P_{mi\,\max} = P_i\left(1+|\Gamma_L|\,|d_i|\right)^2 10^{\Delta C_i/10}$$

$$P_{mr\,\min} = P_i\left(|\Gamma_L|-|d_r|\right)^2 10^{-\Delta C_r/10}$$

$$P_{mr\,\max} = P_i\left(|\Gamma_L|+|d_r|\right)^2 10^{\Delta C_r/10}$$

For other cases, the formulas are obtained analogously and their derivation is left to the reader.

### 6.2.4 Absorbed Power Measurement Uncertainty

The bounds on the power $P_L$ absorbed in the load are obtained as the worst-case differences in the formula $P_L = P_i - P_r$:

$$P_{L\,\min} = P_{mi\,\min} - P_{mr\,\max}$$

$$P_{L\,\max} = P_{mi\,\max} - P_{mr\,\min}$$

## *6.3 Reflection Coefficient Measurement Using Directional Couplers*

Using the equations derived in Section 6.1 for ideal-directivity couplers the load reflection coefficient can be obtained as[9]

$$\Gamma_L = \frac{b_{cr}}{b_{ci}}\frac{c_i}{c_r}$$

The same formula is used also for non-ideal couplers. However, since the true values of the coupling factors are not exactly known; we can only use the nominal values $c_{in}, c_{rn}$. What we obtain is the *measured* value $\Gamma_m$ of the load reflection coefficient:

$$\Gamma_m = \frac{b_{cr}}{b_{ci}}\frac{c_{in}}{c_{rn}} = \frac{c_r}{c_{rn}}\frac{c_{in}}{c_i}\frac{\Gamma_L + d_r}{1+\Gamma_L d_i}$$

As the formula shows, the measured reflection coefficient differs from the actual reflection coefficient $\Gamma_L$. There are three sources of errors:

---

[9] Note that the coefficient $c_i/c_r$ compensates for possible unequal coupling factors of the forward and reverse couplers.

1. *Tracking error* is caused by unequal coupling of the forward and reverse couplers (the multiplicative factor differing from unity). It affects reflection coefficients of all magnitudes and in scalar systems can be reduced (although not quite eliminated) by proper scaling of $|\Gamma_m|$ when measuring e.g. a short circuit.
2. *Directivity error* is caused by the finite reverse coupler directivity (nonzero $d_r$). It is the most significant error since it affects small reflection coefficient measurements, which is the most important practical task. The formula shows that in the absence of the remaining errors the perfect match ($\Gamma_L=0$) would be measured as $d_r$.
3. *Test port mismatch error* is caused by the finite forward coupler directivity (nonzero $d_i$), i.e. its inability to accurately measure the incident power[10]. As seen from the formula, this error is insignificant for small reflection coefficient measurements, because then the denominator approaches unity.

In scalar measurement systems, only the magnitude of reflection coefficient can be measured. Taking the absolute value, the formula yields

$$|\Gamma_m| = \sqrt{\frac{P_{mr}}{P_{mi}}} = \left|\frac{c_r}{c_{rn}}\frac{c_{in}}{c_i}\right|\left|\frac{\Gamma_L + d_r}{1+\Gamma_L d_i}\right|$$

The first equality is in fact the instruction how the reflection coefficient is to be measured. The second equality can be used for the assessment of the measurement error. In fact, since the numerator is the square root of the reflected power and the denominator is the square root of the incident power, the *power* measurement uncertainties elaboraterd in Section 6.2 can be used for the assessment. Pursuing this[11], we obtain

$$|\Gamma_m|_{\min} = \sqrt{\frac{P_{mr\min}}{P_{mi\max}}} = \frac{\left||\Gamma_L|-|d_r|\right|}{1+|\Gamma_L||d_i|}10^{-(\Delta C_i+\Delta C_r)/10}$$

$$|\Gamma_m|_{\max} = \sqrt{\frac{P_{mr\max}}{P_{mi\min}}} = \frac{|\Gamma_L|+|d_r|}{1-|\Gamma_L||d_i|}10^{(\Delta C_i+\Delta C_r)/10}$$

Provided the coupling factors uncertanties are eliminated by short-circuit calibration scaling[12], the formulas simplify to

$$|\Gamma_m|_{\min} = \frac{\left||\Gamma_L|-|d_r|\right|}{1+|\Gamma_L||d_i|}$$

$$|\Gamma_m|_{\max} = \frac{|\Gamma_L|+|d_r|}{1-|\Gamma_L||d_i|}$$

## *6.4 Estimates of True Values*

The formulas derived in this section enable us to find intervals where measured values would lie if the true values were known. In practice, however, the situation is exactly the opposite: we have a measured value and wish to find an interval where the true value very likely can be found. Although the inverse

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[10] The effect is similar as if the coupler were ideal ($d_i$ =0) but there had been a discontinuity in the waveguide between the coupler and the measured load (test port mismatch). The discontinuity causes reflections modifying the incident wave, which cannot be seen by the coupler since it is placed behind. This is the origin of the name of the error.

[11] We are treating the case described as <u>Case Known</u> in Section 6.2. The other cases are analogous and the analysis is left to the reader.

[12] A short circuit (or any totally reflective termination) is connected in place of load and the measured reflection coefficient magnitude is observed. All subsequently measured data are divided by this value.

formulas to those derived can be found, a simpler procedure can be applied. If the found intervals are not very wide (which should be true, otherwise the measurement would be of little use), the formulas can be used conversely: the measured values ($P_{mi}$, $|\Gamma_m|$) are substitued in place of the true values ($P_i$, $|\Gamma_L|$) on the right-hand sides of the equations, and the found limits are used for the estimates of the true values. Rewriting thus, for instance, the equations of Sections 6.2.3 and 6.3, we obtain the worst-case estimates

$$P_{i\,\mathrm{min}} = P_{mi}\left(1-|\Gamma_m||d_i|\right)^2 10^{-\Delta C_i/10}$$

$$P_{i\,\mathrm{max}} = P_{mi}\left(1+|\Gamma_m||d_i|\right)^2 10^{\Delta C_i/10}$$

$$|\Gamma_L|_{\mathrm{min}} = \frac{\left||\Gamma_m|-|d_r|\right|}{1+|\Gamma_m||d_i|}$$

$$|\Gamma_L|_{\mathrm{max}} = \frac{|\Gamma_m|+|d_r|}{1-|\Gamma_m||d_i|}$$

Estimates and uncertainty intervals for VSWR ($S$) and return loss ($R$) are obtained from the reflection coefficient using the conversion formulas:

$$S = \frac{1+|\Gamma_m|}{1-|\Gamma_m|}$$

$$S_{\mathrm{min}} = \frac{1+|\Gamma_L|_{\mathrm{min}}}{1-|\Gamma_L|_{\mathrm{min}}}$$

$$S_{\mathrm{max}} = \frac{1+|\Gamma_L|_{\mathrm{max}}}{1-|\Gamma_L|_{\mathrm{max}}}$$

$$R = -20\log|\Gamma_m|$$

$$R_{\mathrm{min}} = -20\log|\Gamma_L|_{\mathrm{max}}$$

$$R_{\mathrm{max}} = -20\log|\Gamma_L|_{\mathrm{min}}$$

# 7. CONCLUSIONS

This text has been devoted to the analysis of a linear system consisting of a signal source, a lossless transmission medium (waveguide), and a load.

It has been shown that although the power absorbed in load cannot exceed the source available power, the waves travelling in the waveguide and mediating this transmission can carry powers exceding the available power. For gross source and load mismatches, these powers can even be substantially higher than the source available power.

The travelling wave powers depend not only on the magnitudes of the source and load reflection coefficients but also on their phases, and on the interconnecting waveguide length.

Incident power is a measure of the field strength in the waveguide. For gross mismatches at both ends, the system acts like a resonator, and the field inside the waveguide may be very strong despite the low net transmitted power.

Incident power alone is not a proper measure of the power absorbed in the load. Reflected power should be measured as well and the difference of the two should be taken.

The problem of power and reflection coefficient measurement using directional couplers has also been addressed. Formulas have been derived which enable to find worst-case uncertainty intervals of quantities of interest due to finite directivity of measurement couplers and their coupling factor tolerances.

To acquire a quantitative feeling of the matters, the reader is recommended to use the Microwave Power Transmission Calculator program (PowTrans), which enables:

- To model the signal source (e.g. magnetron) and the load (working space).
- To simulate power flows between them, including actual values and limits for arbitrary phases.
- To estimate measurement erors when measuring powers and load reflection coefficient by means of directional couplers.